

For use in exams from the June 2017 Series onwards

## DATA - FUNDAMENTAL CONSTANTS AND VALUES

Quantity	Symbol	Value	Units
speed of light in vacuo	$c$	$3.00 \times 10^8$	$\text{m s}^{-1}$
permeability of free space	$\mu_0$	$4\pi \times 10^{-7}$	$\text{H m}^{-1}$
permittivity of free space	$\epsilon_0$	$8.85 \times 10^{-12}$	$\text{F m}^{-1}$
magnitude of the charge of electron	$e$	$1.60 \times 10^{-19}$	C
the Planck constant	$h$	$6.63 \times 10^{-34}$	J s
gravitational constant	$G$	$6.67 \times 10^{-11}$	$\text{N m}^2 \text{kg}^{-2}$
the Avogadro constant	$N_A$	$6.02 \times 10^{23}$	$\text{mol}^{-1}$
molar gas constant	$R$	8.31	$\text{J K}^{-1} \text{mol}^{-1}$
the Boltzmann constant	$k$	$1.38 \times 10^{-23}$	$\text{J K}^{-1}$
the Stefan constant	$\sigma$	$5.67 \times 10^{-8}$	$\text{W m}^{-2} \text{K}^{-4}$
the Wien constant	$\alpha$	$2.90 \times 10^{-3}$	m K
electron rest mass (equivalent to $5.5 \times 10^{-4}$ u)	$m_e$	$9.11 \times 10^{-31}$	kg
magnitude of electron charge/mass ratio	$\frac{e}{m_e}$	$1.76 \times 10^{11}$	$\text{C kg}^{-1}$
proton rest mass (equivalent to 1.00728 u)	$m_p$	$1.67(3) \times 10^{-27}$	kg
proton charge/mass ratio	$\frac{e}{m_p}$	$9.58 \times 10^7$	$\text{C kg}^{-1}$
neutron rest mass (equivalent to 1.00867 u)	$m_n$	$1.67(5) \times 10^{-27}$	kg
gravitational field strength	$g$	9.81	$\text{N kg}^{-1}$
acceleration due to gravity	$g$	9.81	$\text{m s}^{-2}$
atomic mass unit (1u is equivalent to 931.5 MeV)	u	$1.661 \times 10^{-27}$	kg

### ALGEBRAIC EQUATION

quadratic equation  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

### ASTRONOMICAL DATA

Body	Mass/kg	Mean radius/m
Sun	$1.99 \times 10^{30}$	$6.96 \times 10^8$
Earth	$5.97 \times 10^{24}$	$6.37 \times 10^6$

### GEOMETRICAL EQUATIONS

arc length =  $r\theta$

circumference of circle =  $2\pi r$

area of circle =  $\pi r^2$

curved surface area of cylinder =  $2\pi rh$

area of sphere =  $4\pi r^2$

volume of sphere =  $\frac{4}{3}\pi r^3$

### Particle Physics

Class	Name	Symbol	Rest energy/MeV
photon	photon	$\gamma$	0
lepton	neutrino	$\nu_e$	0
		$\nu_\mu$	0
	electron	$e^\pm$	0.510999
	muon	$\mu^\pm$	105.659
mesons	$\pi$ meson	$\pi^\pm$	139.576
		$\pi^0$	134.972
	K meson	$K^\pm$	493.821
		$K^0$	497.762
baryons	proton	p	938.257
	neutron	n	939.551

### Properties of quarks

antiquarks have opposite signs

Type	Charge	Baryon number	Strangeness
<b>u</b>	$+\frac{2}{3}e$	$+\frac{1}{3}$	0
<b>d</b>	$-\frac{1}{3}e$	$+\frac{1}{3}$	0
<b>s</b>	$-\frac{1}{3}e$	$+\frac{1}{3}$	-1

### Properties of Leptons

	Lepton number
Particles: $e^-, \nu_e; \mu^-, \nu_\mu$	+1
Antiparticles: $e^+, \bar{\nu}_e, \mu^+, \bar{\nu}_\mu$	-1

### Photons and energy levels

photon energy  $E = hf = \frac{hc}{\lambda}$

photoelectricity  $hf = \phi + E_{k(max)}$

energy levels  $hf = E_1 - E_2$

de Broglie wavelength  $\lambda = \frac{h}{p} = \frac{h}{mv}$

### Waves

wave speed  $c = f\lambda$  period  $f = \frac{1}{T}$

first harmonic  $f = \frac{1}{2l} \sqrt{\frac{T}{\mu}}$

fringe spacing  $w = \frac{\lambda D}{s}$  diffraction grating  $d \sin \theta = n\lambda$

refractive index of a substance s,  $n = \frac{c}{c_s}$

for two different substances of refractive indices  $n_1$  and  $n_2$ ,  
law of refraction  $n_1 \sin \theta_1 = n_2 \sin \theta_2$

critical angle  $\sin \theta_c = \frac{n_2}{n_1}$  for  $n_1 > n_2$

### Mechanics

moments moment =  $Fd$

velocity and acceleration  $v = \frac{\Delta s}{\Delta t}$   $a = \frac{\Delta v}{\Delta t}$

equations of motion  $v = u + at$   $s = \left(\frac{u+v}{2}\right)t$

$v^2 = u^2 + 2as$   $s = ut + \frac{at^2}{2}$

force  $F = ma$

force  $F = \frac{\Delta(mv)}{\Delta t}$

impulse  $F \Delta t = \Delta(mv)$

work, energy and power  $W = F s \cos \theta$

$E_k = \frac{1}{2} m v^2$   $\Delta E_p = mg\Delta h$

$P = \frac{\Delta W}{\Delta t}, P = Fv$

efficiency =  $\frac{\text{useful output power}}{\text{input power}}$

### Materials

density  $\rho = \frac{m}{V}$  Hooke's law  $F = k \Delta L$

Young modulus =  $\frac{\text{tensile stress}}{\text{tensile strain}}$  tensile stress =  $\frac{F}{A}$

tensile strain =  $\frac{\Delta L}{L}$

energy stored  $E = \frac{1}{2} F \Delta L$

## Electricity

$$\text{current and pd} \quad I = \frac{\Delta Q}{\Delta t} \quad V = \frac{W}{Q} \quad R = \frac{V}{I}$$

$$\text{resistivity} \quad \rho = \frac{RA}{L}$$

$$\text{resistors in series} \quad R_T = R_1 + R_2 + R_3 + \dots$$

$$\text{resistors in parallel} \quad \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

$$\text{power} \quad P = VI = I^2R = \frac{V^2}{R}$$

$$\text{emf} \quad \varepsilon = \frac{E}{Q} \quad \varepsilon = I(R + r)$$

## Circular motion

$$\text{magnitude of angular speed} \quad \omega = \frac{v}{r}$$

$$\omega = 2\pi f$$

$$\text{centripetal acceleration} \quad a = \frac{v^2}{r} = \omega^2 r$$

$$\text{centripetal force} \quad F = \frac{mv^2}{r} = m\omega^2 r$$

## Simple harmonic motion

$$\text{acceleration} \quad a = -\omega^2 x$$

$$\text{displacement} \quad x = A \cos(\omega t)$$

$$\text{speed} \quad v = \pm \omega \sqrt{(A^2 - x^2)}$$

$$\text{maximum speed} \quad v_{\max} = \omega A$$

$$\text{maximum acceleration} \quad a_{\max} = \omega^2 A$$

$$\text{for a mass-spring system} \quad T = 2\pi \sqrt{\frac{m}{k}}$$

$$\text{for a simple pendulum} \quad T = 2\pi \sqrt{\frac{l}{g}}$$

## Thermal physics

$$\text{energy to change temperature} \quad Q = mc\Delta\theta$$

$$\text{energy to change state} \quad Q = ml$$

$$\text{gas law} \quad pV = nRT \\ pV = NkT$$

$$\text{kinetic theory model} \quad pV = \frac{1}{3}Nm(c_{\text{rms}})^2$$

$$\text{kinetic energy of gas molecule} \quad \frac{1}{2}m(c_{\text{rms}})^2 = \frac{3}{2}kT = \frac{3RT}{2N_A}$$

## Gravitational fields

$$\text{force between two masses} \quad F = \frac{Gm_1m_2}{r^2}$$

$$\text{gravitational field strength} \quad g = \frac{F}{m}$$

$$\text{magnitude of gravitational field strength in a radial field} \quad g = \frac{GM}{r^2}$$

$$\text{work done} \quad \Delta W = m\Delta V$$

$$\text{gravitational potential} \quad V = -\frac{GM}{r}$$

$$g = -\frac{\Delta V}{\Delta r}$$

## Electric fields and capacitors

$$\text{force between two point charges} \quad F = \frac{1}{4\pi\epsilon_0} \frac{Q_1Q_2}{r^2}$$

$$\text{force on a charge} \quad F = EQ$$

$$\text{field strength for a uniform field} \quad E = \frac{V}{d}$$

$$\text{work done} \quad \Delta W = Q\Delta V$$

$$\text{field strength for a radial field} \quad E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$\text{electric potential} \quad V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

$$\text{field strength} \quad E = \frac{\Delta V}{\Delta r}$$

$$\text{capacitance} \quad C = \frac{Q}{V}$$

$$C = \frac{A\epsilon_0\epsilon_r}{d}$$

$$\text{capacitor energy stored} \quad E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2} \frac{Q^2}{C}$$

$$\text{capacitor charging} \quad Q = Q_0(1 - e^{-\frac{t}{RC}})$$

$$\text{decay of charge} \quad Q = Q_0 e^{-\frac{t}{RC}}$$

$$\text{time constant} \quad RC$$

## Magnetic fields

<i>force on a current</i>	$F = BIl$
<i>force on a moving charge</i>	$F = BQv$
<i>magnetic flux</i>	$\Phi = BA$
<i>magnetic flux linkage</i>	$N\Phi = BAN \cos \theta$
<i>magnitude of induced emf</i>	$\varepsilon = N \frac{\Delta\Phi}{\Delta t}$
	$N\Phi = BAN \cos \theta$
<i>emf induced in a rotating coil</i>	$\varepsilon = BAN\omega \sin \omega t$
<i>alternating current</i>	$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \quad V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$
<i>transformer equations</i>	$\frac{N_s}{N_p} = \frac{V_s}{V_p}$
	$\text{efficiency} = \frac{I_s V_s}{I_p V_p}$

## Nuclear physics

<i>inverse square law for <math>\gamma</math> radiation</i>	$I = \frac{k}{x^2}$
<i>radioactive decay</i>	$\frac{\Delta N}{\Delta t} = -\lambda N, N = N_0 e^{-\lambda t}$
<i>activity</i>	$A = \lambda N$
<i>half-life</i>	$T_{1/2} = \frac{\ln 2}{\lambda}$
<i>nuclear radius</i>	$R = R_0 A^{1/3}$
<i>energy-mass equation</i>	$E = mc^2$

## OPTIONS

### Astrophysics

1 astronomical unit	$= 1.50 \times 10^{11} \text{ m}$
1 light year	$= 9.46 \times 10^{15} \text{ m}$
1 parsec	$= 2.06 \times 10^5 \text{ AU} = 3.08 \times 10^{16} \text{ m}$ $= 3.26 \text{ ly}$
Hubble constant, $H$	$= 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$
$M = \frac{\text{angle subtended by image at eye}}{\text{angle subtended by object at unaided eye}}$	
<i>telescope in normal adjustment</i>	$M = \frac{f_o}{f_e}$
<i>Rayleigh criterion</i>	$\theta \approx \frac{\lambda}{D}$
<i>magnitude equation</i>	$m - M = 5 \log \frac{d}{10}$
<i>Wien's law</i>	$\lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ m K}$
<i>Stefan's law</i>	$P = \sigma AT^4$
<i>Schwarzschild radius</i>	$R_s \approx \frac{2GM}{c^2}$
<i>Doppler shift for <math>v \ll c</math></i>	$\frac{\Delta f}{f} = -\frac{\Delta \lambda}{\lambda} = \frac{v}{c}$
<i>red shift</i>	$z = -\frac{v}{c}$
<i>Hubble's law</i>	$v = Hd$

### Medical physics

<i>lens equations</i>	$P = \frac{1}{f}$ $m = \frac{v}{u}$ $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$
<i>threshold of hearing</i>	$I_0 = 1.0 \times 10^{-12} \text{ W m}^{-2}$
<i>intensity level</i>	$\text{intensity level} = 10 \log \frac{I}{I_0}$
<i>absorption</i>	$I = I_0 e^{-\mu x}$ $\mu_m = \frac{\mu}{\rho}$
<i>ultrasound imaging</i>	$Z = p c$ $\frac{I_r}{I_i} = \left( \frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2$
<i>half-lives</i>	$\frac{1}{T_E} = \frac{1}{T_B} + \frac{1}{T_P}$

## Engineering physics

moment of inertia  $I = \Sigma mr^2$

angular kinetic energy  $E_k = \frac{1}{2} I \omega^2$

equations of angular motion

$$\omega_2 = \omega_1 + \alpha t$$

$$\omega_2^2 = \omega_1^2 + 2\alpha\theta$$

$$\theta = \omega_1 t + \frac{\alpha t^2}{2}$$

$$\theta = \frac{(\omega_1 + \omega_2) t}{2}$$

torque

$$T = I \alpha$$

$$T = F r$$

angular momentum

angular momentum =  $I\omega$

angular impulse

$$T\Delta t = \Delta(I\omega)$$

work done

$$W = T\theta$$

power

$$P = T\omega$$

thermodynamics

$$Q = \Delta U + W$$

$$W = p\Delta V$$

adiabatic change

$$pV^\gamma = \text{constant}$$

isothermal change

$$pV = \text{constant}$$

heat engines

$$\text{efficiency} = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$$

$$\text{maximum theoretical efficiency} = \frac{T_H - T_C}{T_H}$$

work done per cycle = area of loop

input power = calorific value  $\times$  fuel flow rate

$$\text{indicated power} = (\text{area of } p - V \text{ loop}) \times (\text{number of cycles per second}) \times (\text{number of cylinders})$$

output or brake power  $P = T\omega$

friction power = indicated power - brake power

heat pumps and refrigerators

$$\text{refrigerator: } COP_{\text{ref}} = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C}$$

$$\text{heat pump: } COP_{\text{hp}} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C}$$

## Turning points in physics

electrons in fields  $F = \frac{eV}{d}$

$$F = Bev$$

$$r = \frac{mv}{Be}$$

$$\frac{1}{2} mv^2 = eV$$

Millikan's experiment  $\frac{QV}{d} = mg$

$$F = 6\pi\eta r v$$

Maxwell's formula

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$$

special relativity

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

## Electronics

resonant frequency

$$f_0 = \frac{1}{2\pi \sqrt{LC}}$$

Q-factor

$$Q = \frac{f_0}{f_B}$$

operational amplifiers: open loop

$$V_{\text{out}} = A_{\text{OL}}(V_+ - V_-)$$

inverting amplifier

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_f}{R_{\text{in}}}$$

non-inverting amplifier

$$\frac{V_{\text{out}}}{V_{\text{in}}} = 1 + \frac{R_f}{R_1}$$

summing amplifier

$$V_{\text{out}} = -R_f \left( \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \dots \right)$$

difference amplifier

$$V_{\text{out}} = (V_+ - V_-) \frac{R_f}{R_1}$$

Bandwidth requirement:

for AM

$$\text{bandwidth} = 2f_M$$

for FM

$$\text{bandwidth} = 2(\Delta f + f_M)$$





